Chapter 13: Equations

Equation 13.1:

$$\begin{aligned} y_i &= \alpha + \beta_1 x_{1i} + \beta_2 (0) + \varepsilon_i \\ &= \alpha + \beta_1 x_{1i} + \varepsilon_i \end{aligned}$$

Equation 13.2:

$$\begin{aligned} y_i &= \alpha + \beta_1 x_{1i} + \beta_2 (1) + \varepsilon_i \\ &= (\alpha + \beta_2) + \beta_1 x_{1i} + \varepsilon_i \end{aligned}$$

Equation 13.3:

$$y_i = \alpha + \beta_2$$

Equation 13.4:

$$\mathrm{E}(y_i \mid x_{2i} = 1)$$

Equation 13.5:

$$\mathrm{E}(y_i \mid x_{2i} = 1) = \mathrm{E}\left((\alpha + \beta_2) + \beta_1 x_{1i} + \varepsilon_i\right)$$

Equation 13.6:

$$\mathrm{E}(y_i \mid x_{2i} = 1) = (\alpha + \beta_2) + \beta_1 x_{1i}$$

Equation 13.7:

$$\mathrm{E}\left(y_i \mid x_{2i}=0\right)$$

Equation 13.8:

$$\mathrm{E}\left(y_i \mid x_{2i}=0\right)=\alpha+\beta_1 x_{1i}$$

Equation 13.9:

$$\begin{aligned}\mathrm{E}\left(y_i \mid x_{2i}=1\right)-\mathrm{E}\left(y_i \mid x_{2i}=0\right) &= \left(\left(\alpha+\beta_2\right)+\beta_1 x_{1i}\right)-\left(\alpha+\beta_1 x_{1i}\right) \\ &= \beta_2\end{aligned}$$

Equation 13.10:

$$x_{1i}=1-x_{2i}$$

Equation 13.11:

$$\mathrm{E}\left(y_i\right)+\Delta y=\alpha+\beta_1\left(x_{1i}+\Delta x\right)+\beta_2 x_{2i}=\alpha+\beta_1 x_{1i}+\beta_2 x_{2i}+\beta_1 \Delta x$$

Equation 13.12:

$$\frac{\Delta y}{\Delta x}=\beta_1$$

Equation 13.13:

$$y_i=\alpha+\beta_1 x_i+\beta_2 x_i^2+\varepsilon_i$$

Equation 13.14:

$$\mathrm{E}\left(y_i\right) = \alpha + \beta_1 x_i + \beta_2 x_i^2$$

Equation 13.15:

$$\begin{aligned}\mathrm{E}\left(y_i\right) + \Delta y &= \alpha + \beta_1\left(x_i + \Delta x\right) + \beta_2\left(x + \Delta x\right)^2 \\ &= \alpha + \beta_1 x_i + \beta_1 \Delta x + \beta_2 x_i^2 + 2\beta_2 x_i \Delta x + \beta_2\left(\Delta x\right)^2\end{aligned}$$

Equation 13.16:

$$\Delta y = \beta_1 \Delta x + 2\beta_2 x_i \Delta x + \beta_2\left(\Delta x\right)^2$$

Equation 13.17:

$$\frac{\Delta y}{\Delta x} \approx \beta_i + 2\beta_2 x_i$$

Equation 13.18:

$$x_i \approx \frac{-\beta_1}{2\beta_2}$$

Equation 13.19:

$$y_i = a + b_1 x_i + b_2 x_i^2 + e_i$$

Equation 13.20:

$$\begin{gathered}\text{earnings} = -50,795 + 3,798(\text{age}) - 40.68(\text{age squared}) + \text{error} \\ (14,629) \quad (748.6) \qquad (9.044)\end{gathered}$$

Equation 13.21:

$$\frac{\Delta y}{\Delta x} \approx b_1 + 2b_2 x_i$$

Equation 13.22:

$$x_i = \frac{-b_1}{2b_2}$$

Equation 13.23:

$$\begin{gathered}\text{earnings} = 19,419 - 4,233(\text{years of schooling}) + 376.2(\text{years of schooling squared}) + \text{error} \\ (6,561) \ (1,110) \qquad\qquad (49.37)\end{gathered}$$

Equation 13.24:

$$y_i = \alpha + \beta \log x_i + \varepsilon_i$$

Equation 13.25:

$$\ln\left(x_i\left(1+\frac{\Delta x}{x_i}\right)\right) = \ln(x_i) + \ln\left(1+\frac{\Delta x}{x_i}\right)$$

Equation 13.26:

$$\ln\left(1+\frac{\Delta x}{x_i}\right)\approx\frac{\Delta x}{x_i}$$

Equation 13.27:

$$y_i=\alpha+\beta\ln x_i+\varepsilon_i$$

Equation 13.28:

$$\mathrm{E}\left(y_i\right)=\alpha+\beta\ln x_i$$

Equation 13.29:

$$\mathrm{E}\left(y_i\right)+\Delta y=\alpha+\beta\ln\left(x_i+\Delta x\right)$$

Equation 13.30:

$$\ln\left(x_i+\Delta x\right)\approx\ln\left(x_i\right)+\frac{\Delta x}{x_i}$$

Equation 13.31:

$$\mathrm{E}\left(y_i\right)+\Delta y\approx\alpha+\beta\ln x_i+\beta\frac{\Delta x}{x_i}$$

Equation 13.32:

$$\Delta y\approx\beta\frac{\Delta x}{x_i}$$

Equation 13.33:

$$\beta \approx \frac{\Delta y}{\Delta x / x_i}$$

Equation 13.34:

$$\begin{gathered}\text{GNI per capita} = -13,421 + 17,257 \ln(\text{CPI}) + e_i \\ (2,183) \quad (1,494)\end{gathered}$$

Equation 13.35:

$$\ln y_i = \alpha + \beta x_i + \varepsilon_i$$

Equation 13.36:

$$\beta = \frac{\text{E}[\Delta y / y_i]}{\Delta x}$$

Equation 13.37:

$$\begin{gathered}\ln(\text{earnings}) = 8.764 + .1055(\text{years of schooling}) + \text{error} \\ (.1305)(.00985)\end{gathered}$$

Equation 13.38:

$$\ln y_i = \alpha + \beta \ln x_i + \varepsilon_i$$

Equation 13.39:

$$\beta = \frac{\mathrm{E}[\Delta y / y_i]}{\Delta x / x_i} = \eta_{yx}$$

Equation 13.40:

$$\ln y_i = a + b \ln x_i + e_i$$

Equation 13.41:

$$\ln(\text{child mortality}) = \underset{(.7216)}{9.761} - \underset{(.1722)}{1.434} \ln\left(\begin{array}{l}\text{percentage of rural population with} \\ \text{access to improved water}\end{array}\right) + e_i$$

Equation 13.42:

$$\mathrm{E}(y_i \mid \text{man, not black}) = \alpha + \beta_1(0) + \beta_2(0) = \alpha$$

Equation 13.43:

$$\mathrm{E}(y_i \mid \text{woman, not black}) = \alpha + \beta_1(1) + \beta_2(0) = \alpha + \beta_1$$

Equation 13.44:

$$\mathrm{E}(y_i \mid \text{man, black}) = \alpha + \beta_1(0) + \beta_2(1) = \alpha + \beta_2$$

Equation 13.45:

$$\mathrm{E}(y_i \mid \text{woman, black}) = \alpha + \beta_1(1) + \beta_2(1) = \alpha + \beta_1 + \beta_2$$

Equation 13.46:

$$\mathrm{E}\left(y_i \mid \text{woman, not black}\right) - \mathrm{E}\left(y_i \mid \text{man, not black}\right) = \left(\alpha + \beta_1\right) - \alpha = \beta_1$$

Equation 13.47:

$$\mathrm{E}\left(y_i \mid \text{woman, black}\right) - \mathrm{E}\left(y_i \mid \text{man, black}\right) = \left(\alpha + \beta_1 + \beta_2\right) - \left(\alpha + \beta_2\right) = \beta_1$$

Equation 13.48:

$$y_i = \alpha + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{1i} x_{2i} + \varepsilon_i$$

Equation 13.49:

$$\mathrm{E}\left(y_i\right) = \alpha + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{1i} x_{2i}.$$

Equation 13.50:

$$\begin{aligned}\mathrm{E}\left(y_i\right) + \Delta y &= \alpha + \beta_1\left(x_{1i} + \Delta x_1\right) + \beta_2 x_{2i} + \beta_3\left(x_{1i} + \Delta x_1\right) x_{2i} \\ &= \alpha + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{1i} x_{2i} + \beta_1 \Delta x_1 + \beta_3 x_{2i} \Delta x_1\end{aligned}$$

Equation 13.51:

$$\Delta y = \beta_1 \Delta x_1 + \beta_3 x_{2i} \Delta x_1$$

Equation 13.52:

$$\frac{\Delta y}{\Delta x_1} = \beta_1 + \beta_3 x_{2i}$$

Equation 13.53:

$$\mathrm{E}\left(y_i \mid \text{man, not black}\right) = \alpha + \beta_1(0) + \beta_2(0) + \beta_3(0) = \alpha$$

Equation 13.54:

$$\mathrm{E}\left(y_i \mid \text{woman, not black}\right) = \alpha + \beta_1(1) + \beta_2(0) + \beta_3(0) = \alpha + \beta_1$$

Equation 13.55:

$$\mathrm{E}\left(y_i \mid \text{man, black}\right) = \alpha + \beta_1(0) + \beta_2(1) + \beta_3(0) = \alpha + \beta_2$$

Equation 13.56:

$$\mathrm{E}\left(y_i \mid \text{woman, black}\right) = \alpha + \beta_1(1) + \beta_2(1) + \beta_3(1) = \alpha + \beta_1 + \beta_2 + \beta_3$$

Equation 13.57:

$$\mathrm{E}\left(y_i \mid \text{woman, black}\right) - \mathrm{E}\left(y_i \mid \text{man, black}\right) = \left(\alpha + \beta_1 + \beta_2 + \beta_3\right) - \left(\alpha + \beta_2\right) = \beta_1 + \beta_3$$

Equation 13.58:

$$y_i = a + b_1 x_{1i} + b_2 x_{2i} + b_3 x_{1i} x_{2i} + e_i$$

Equation 13.59:

$$\begin{gathered}\text{earnings} = 40,060 - 18,612\left(\text{female}\right) - 13,163\left(\text{black}\right) + 11,561\left(\text{black female}\right) + \text{error} \\ \left(1,923\right)\ \ \left(2,740\right) \qquad\qquad \left(7,317\right) \qquad\quad \left(10,354\right)\end{gathered}$$

Equation 13.60:

$$\underset{\quad}{\text{earnings} = 40,784 - 19,581(\text{female}) - 14,520(\text{black}) + 14,352(\text{black female}) + \text{error}}$$
$$(153.4)\quad (218.7)\qquad\qquad (620.2)\qquad\qquad (870.1)$$

Equation 13.61:

$$\begin{aligned}\text{earnings} = {} & -21,541 + 10,207(\text{female}) + 4,841(\text{years of schooling}) \\ & (6,098)\quad (8,887)\qquad\qquad (466.1) \\ & -2,226(\text{years of schooling for women}) + \text{error} \\ & (682.0)\end{aligned}$$

Equation 13.62:

$$\begin{aligned}\text{earnings} = {} & 7,678 - 552.5(\text{age}) + 582.2(\text{years of schooling}) \\ & (17,270)\,(402.0)\qquad (1,366) \\ & +74.23(\text{age} * \text{years of schooling}) + \text{error} \\ & (31.37)\end{aligned}$$

Equation 13.63:

$$\begin{aligned}\text{earnings} = {} & -7,678 - 201.1(\text{age}) + 2,079(\text{years of schooling}) \\ & (1,232)\quad (28.64)\qquad\qquad (98.60) \\ & +39.99(\text{age} * \text{years of schooling}) + \text{error} \\ & (2.259)\end{aligned}$$

Equation 13.64:

$$\frac{\Delta(\text{earnings})}{\Delta(\text{age})} = -201.1 + 39.99(\text{years of schooling})$$